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Employing controlled vibrations to predict fluid forces on a cylinder undergoing vortex-induced vibration

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Abstract

In the present study, we measure the fluid forces on a vertical cylinder that is forced to vibrate transversely to a water channel flow, and compare directly to the forces encountered by freely vibrating cylinders, under conditions where we carefully match the amplitude, frequency, and Reynolds number (Re) of the two cases. A key point is that we use precisely the same cylinder and submerged flow configuration for both the free and controlled cases. Where the free vibration exhibits closely sinusoidal motion, the controlled sinusoidal motion yields forces in close agreement with the free vibration case. Although this result might be expected, previous comparisons have not been uniformly close, which highlights the importance of matching the experimental conditions precisely, and of accurately measuring the phase between the force and body motion. For a lightly damped system, which is perhaps the most significant case to analyze, one typically finds that the maximum response amplitude is quite unsteady. One might conventionally expect prediction of forces to be difficult in such cases. However, it is of practical significance that, even in this case, a quasi-steady approximation is effective. This is a significant point because it suggests that controlled vibration measurements for constant amplitude motion might remain applicable to free vibration systems undergoing even transient or intermittent motions.

Keywords: Vortex-induced vibration; Oscillating cylinder

1. Introduction

The problem of vortex-induced vibration is of interest to many fields of engineering. It affects, for example, the dynamics of riser tubes bringing oil from the seabed to the surface, the flow around heat exchanger tubes, and the design of civil engineering structures such as bridges and chimneys. An overview of recent phenomena in vortex-induced vibration can be found in the review by Williamson and Govardhan (2004).

The case of an elastically mounted rigid cylinder that is confined to move transversely to the flow is often used as a paradigm for understanding the problem of vortex-induced vibration in general. Such a system, for low mass and damping, has been shown to have three branches of response as the normalized velocity is increased: an initial branch, upper branch, and lower branch, with a hysteresis between the initial and upper branches, and intermittent switching between the upper and lower branches (Khalak and Williamson, 1999). A central question in the study of

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vortex-induced vibration of an elastically mounted (free vibration) cylinder is to what extent can results from controlled vibration experiments, where the cylinder is prescribed to have a sinusoidal motion, be applied to the case where the cylinder oscillates freely under vortex-induced motion.

There have been many studies of controlled vibrations of cylinders. For example, Mercier (1973), Sarpkaya (1977), and Carberry et al. (2001, 2005) measured the forces on a cylinder that is controlled to oscillate with constant amplitude, over a range of frequencies. Carberry et al. also used digital particle image velocimetry (DPIV) to examine the wake vortex dynamics. Staubli (1983) and Gopalkrishnan (1993) made force measurements over a range of amplitudes and frequencies. However, very few past investigations have focused on direct comparisons of fluid forces between the two cases of controlled vibration and free vibration, *using precisely the same experimental arrangements* for the submerged cylinder, except for the recent works of Triantafyllou, Hover and co-workers at MIT using their Virtual Cable Testing Apparatus in a towing tank (Hover et al., 1998). In the present study, we have carefully matched the experimental arrangement between free and controlled vibration cases, which appears to be important if one wants to accurately compare predicted and measured responses for freely vibrating systems.

For an elastically mounted cylinder, when the body oscillation frequency is synchronized with the periodic vortex mode, the force, $F(t)$, and the response displacement, $y(t)$, are often approximated by the following representations:

$$F(t) = F_0 \sin(2\pi f t + \phi), \tag{1}$$

$$y(t) = A \sin(2\pi f t). \tag{2}$$

The phase angle, ϕ, between the fluid force and the body displacement is crucial in determining the energy transfer from the fluid to the body, which in this simplified case, is given by

$$E = \pi A F_0 \sin \phi. \tag{3}$$

This energy input is balanced by the energy dissipated due to the structural damping. Therefore, for free vibration to occur, the phase angle must lie in the range $\phi = 0$–180°. For very low mass-damping, (which leads to the highest peak amplitude), the energy dissipated is very low, and thus the phase is close to 0°, or close to 180°. This presents a difficulty in accurately predicting free vibration at low mass-damping with controlled vibration experiments, because a small difference in phase angle of just say 2–4° can cause the system to change from positive to negative excitation.

2. Experimental details

The present experiments were conducted in the Cornell-ONR Water Channel, which has a cross-section of 38.1 cm × 50.8 cm. The turbulence level in the test-section of the water channel was less than 0.9% over the range of velocities (5–30 cm/s) used in this study. A cylinder of diameter 3.81 cm and length 38.1 cm was suspended vertically in the water channel and forced to oscillate transverse to the flow using a computer-controlled motor attached to a transverse lead screw. A fixed end-plate was placed 2 mm below the bottom of the cylinder (but not in contact with the cylinder) to encourage two-dimensional vortex shedding, following the study of Khalak and Williamson (1996). The range of Re in this study was 2400–6800.

A two-axis force balance utilizing linear variable distance transducers (LVDTs) was used to measure the lift and drag forces on the cylinder. The transverse displacement of the cylinder was measured using a non-contact (magnetostrictive) position transducer. The measured force signals were filtered using a low-pass filter at 5 Hz. The small phase lag due to filtering (typically 3–4°) was carefully measured, and the force signals were systematically adjusted to account for it. Also, the inertial forces in the transverse direction were subtracted from the total measured force. Instantaneous phase information was obtained through use of the Hilbert transform, where we follow closely the details described in Khalak and Williamson (1996).

3. Direct comparison to free vibration

In our experiments, we chose to directly match the values of $A^*, U^*/f^*$, and Re at several points in the initial, upper, and lower amplitude branches of the free vibration response of Govardhan and Williamson (2000), shown in Fig. 1(a). (In this study, $A^* = A/D =$ amplitude/diameter; $U^*/f^* = U/fD$, where $U =$ free stream velocity, $f =$ oscillation frequency). The measured transverse force coefficient for controlled vibrations agrees very well with the free vibration data in all three response branches. The phase angle also agrees well in all three branches, but is very close

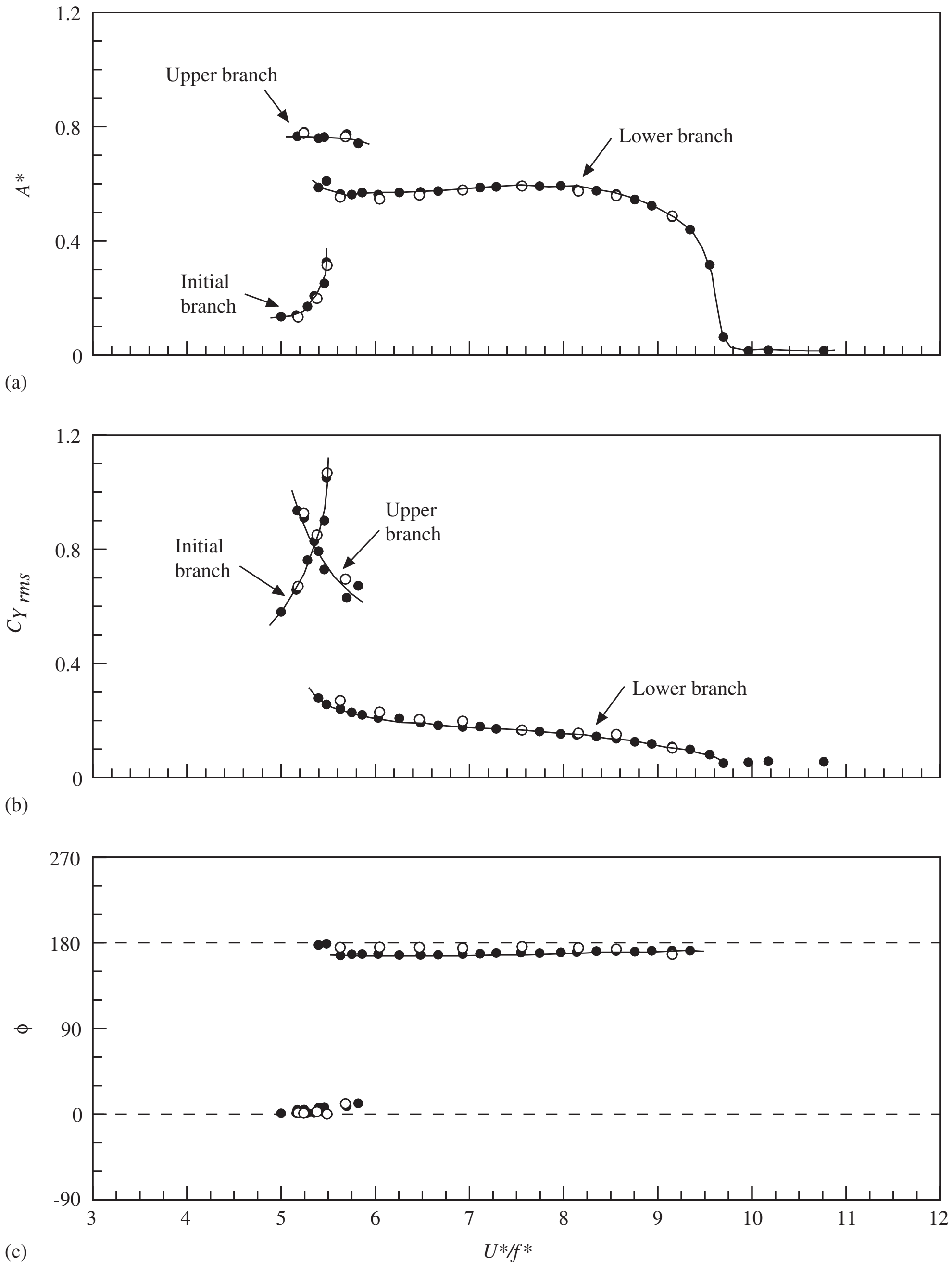

Fig. 1. Direct comparison of controlled and free vibration at the matched values of $A^*, U^*/f^*$, Re: •, free vibration, $m^* = 8.6$, $\zeta = 0.0016$ (Govardhan and Williamson, 2000); ○, sinusoidal controlled vibration (present results). Re = 2400–6800.

to 0° in the initial and upper branches, and very close to 180° in the lower branch. This illustrates the sensitive nature of measuring the phase angle to predict the correct sign of energy transfer.

We might expect good agreement between sinusoidal controlled vibration and free vibration in the lower branch, where the free vibration response is close to sinusoidal. We show, in Fig. 2, time traces of position, force, and phase, from free vibration in the lower branch, and the corresponding time traces from a controlled vibration experiment at the same amplitude and frequency. The free vibration motion is so periodic that it is difficult to tell which time trace is

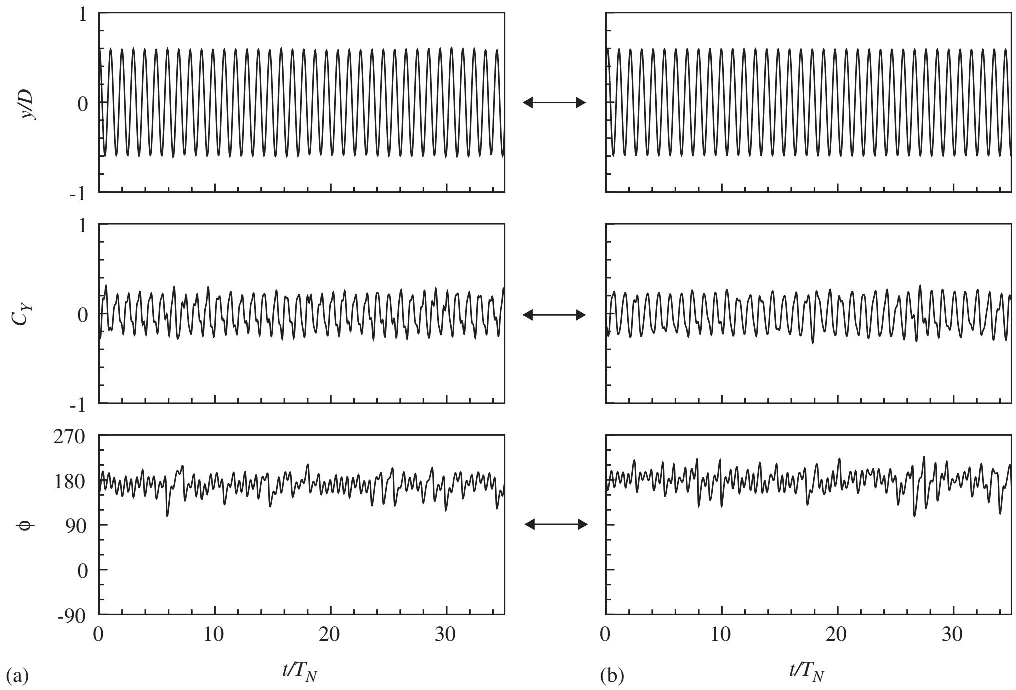

Fig. 2. Comparison of the position, force coefficient, and phase time traces in the lower branch, $U^*/f^* = 7.55$, for both (a) free vibration and (b) controlled vibration.

controlled and which is free. However, one might ask how successful is controlled vibration in regions of intermittent switching, where the response is known to jump between the upper branch and the lower branch (Khalak and Williamson, 1999). This is an important question if we want to be able to accurately predict peak amplitudes (which are especially of interest to practicing engineers) since the peak amplitude in free vibration occurs where there can exist intermittent mode switching, for systems of low mass and damping.

We focus on the regime of intermittent switching in Fig. 3. Our approach here is to treat the motion as quasi-steady. We matched the amplitude and frequency of the upper branch, and separately matched the amplitude and frequency of the lower branch, using sinusoidal controlled vibrations. The intriguing result is that even in regions of intermittent switching, where the cylinder is jumping between modes, sinusoidal controlled vibration reasonably represents the force and phase angle of free vibration during periods when the system resides in one state or the other. This suggests that one might use controlled vibration results to accurately predict the peak amplitude of a freely vibrating body, even in the presence of unsteady vibrations.

4. Controlled vibration at constant amplitude

In addition to comparing with free vibration results, we also performed controlled vibration experiments to compare directly with the controlled vibration results of Carberry et al. (2005) for constant values of amplitude. We matched $A^* = 0.5 =$ constant, $\mathrm{Re} = 4400$ and varied (U^*/f^*), as in their study. In order to relate such a constant amplitude-controlled vibration experiment to free vibration, we plot our chosen amplitude conditions in Fig. 4(a) and compare with a typical free vibration response plot. We find a phase angle below 180° for controlled vibration over the regime of $(U^*/f^*) = 5.5$–9.5, corresponding to the lower branch, yielding positive energy transfer, which is consistent with the fact that the free vibration response exists in this regime. It is particularly interesting that the phase exceeds 180° at $(U^*/f^*) = 9.5$, suggesting a switch from positive to negative excitation, at a point which is roughly where the free vibration becomes desynchronized and where the amplitude, A^*, falls below 0.5. We also see a jump in the force and phase at $(U^*/f^*) = 5.6$, which corresponds to where a mode transition is seen in the free vibration response. The sudden

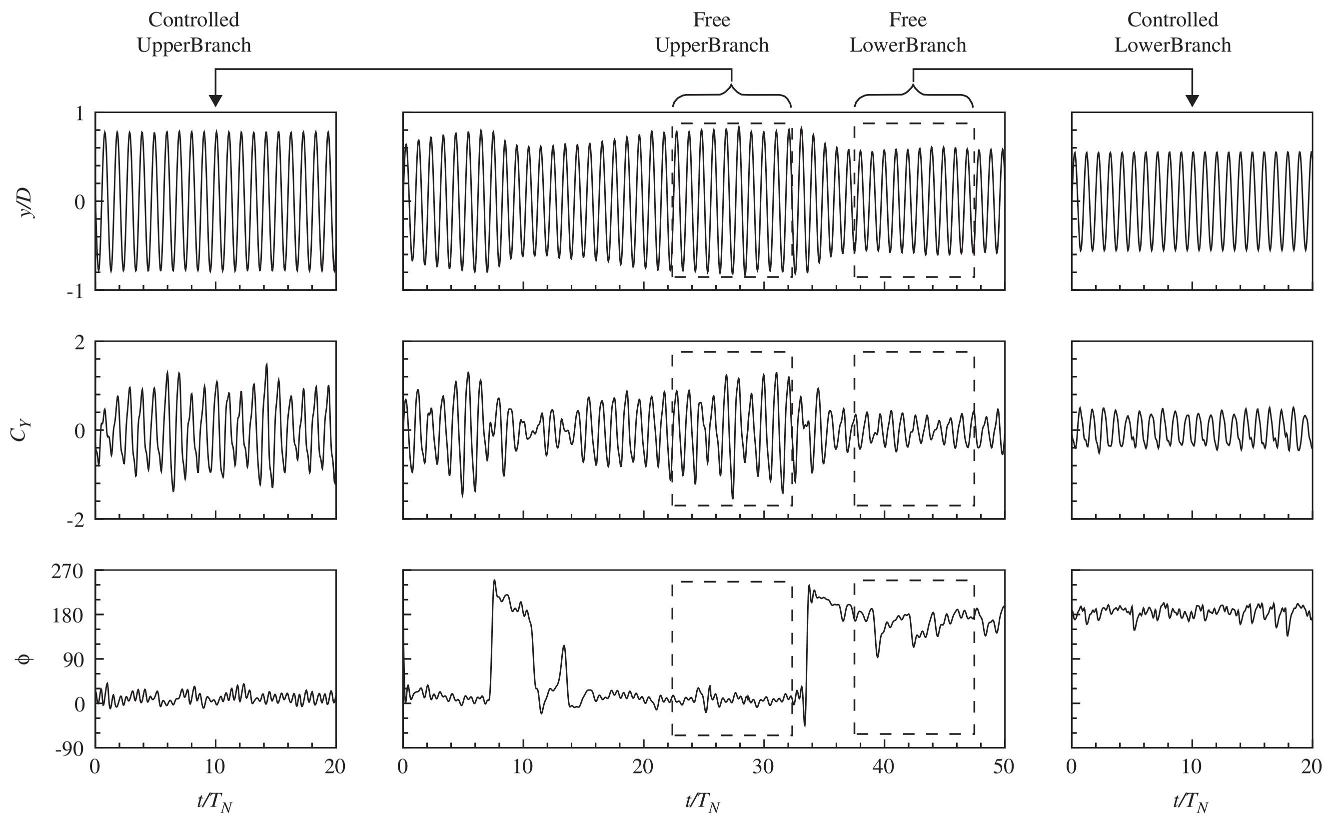

Fig. 3. The position, force coefficient, and phase angle time traces in the intermittent switching region of the free vibration response, $U^*/f^* = 5.6$. By matching the upper branch free response conditions with sinusoidal controlled vibration, we find reasonable agreement of the force and phase angle. Separate matching for the lower branch yields similarly good agreement of force and phase.

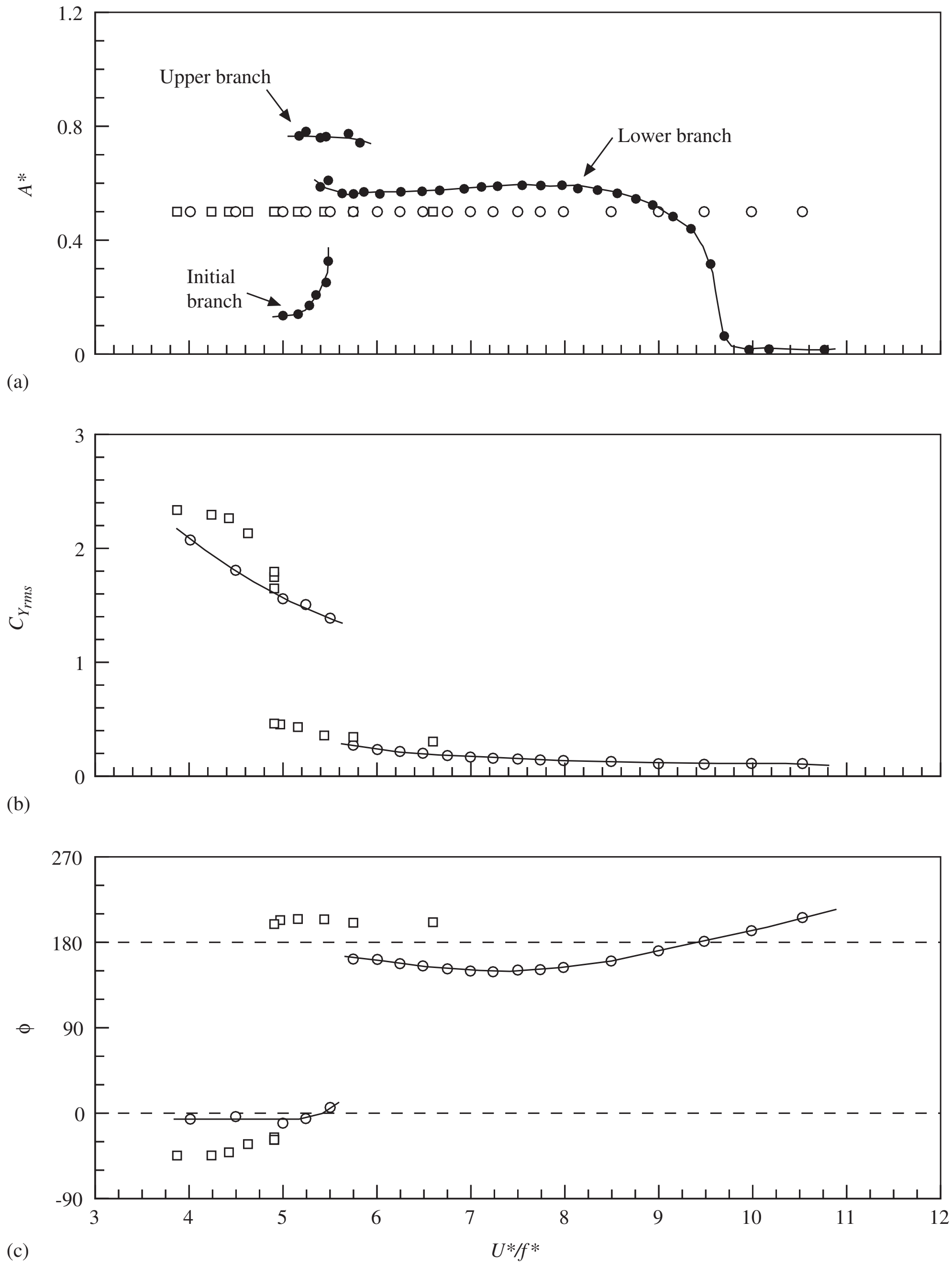

Fig. 4. Controlled sinusoidal vibrations at constant $A^* = 0.5$, Re $= 4400$: $\bullet$, free vibration response plot, $m^* = 8.6$, $\zeta = 0.0016$ (Govardhan and Williamson, 2000); $\circ$, sinusoidal controlled vibration (present results); $\square$, sinusoidal controlled vibration from Carberry et al. (2005).

switch to negative energy transfer at this point would suggest that free vibration would not exist for $(U^*/f^*) < 5.6$, which is what one finds for elastically mounted bodies at this level of amplitude, $A^* = 0.5$.

We find excellent agreement in the measurement of the force coefficient between our data and those of Carberry et al. (2005), in the lower branch of response. The fact that Carberry et al. find negative energy transfer over part of this lower branch regime, where free vibrations are found in previous studies, could perhaps be related to quite different experimental arrangements. While in our set-up the cylinder was suspended vertically in the water channel and oscillated above a fixed end-plate, Carberry et al. supported the cylinder horizontally with end-plates that moved with the cylinder. It is possible that differences in experimental arrangements would influence the measurement of force and phase.

5. Conclusions

Our controlled vibrations of a cylinder are arranged to carefully match the amplitude, frequency, and Reynolds number of an elastically mounted cylinder at low mass and damping. Despite the fact that one might naturally expect good agreement between the forces if the free and controlled vibrations are both close to sinusoidal, since the body effectively follows almost the same path through the fluid, previous comparisons have not necessarily been close. We may conclude that it is important to carefully match the experimental conditions between free and controlled vibration, such as in the present study, or in the experiments of Hover et al. (1998), to accurately predict forces applicable to free vibration. At low mass and damping, the excitation needed to balance the energy dissipation is small, and so the phase of the fluid force is also small, thus the precise measurement of phase is very important.

In cases of very low mass and damping, the peak amplitude response of a lightly damped body is quite unsteady; the cylinder is subject to an intermittent switching between an upper and a lower response amplitude branch, where close comparison with forced vibration results is more difficult. However, it is of practical significance that even in this case, a quasi-steady approximation is effective. For example, if one vibrates the body in a controlled steady state oscillation corresponding to the upper branch conditions, the forces compare well with such measurements taken over intermittent time periods when the free system resides in the upper response branch. The same is true of the lower branch response conditions. This is an important point because it suggests that controlled vibration measurements for constant amplitude motion might remain applicable, in a *quasi-steady manner*, to free vibration systems undergoing even transient motions with unsteady amplitudes. In our further work, it also appears that this quasi-steady approach will yield insight into the mode jumps that occur as the freely vibrating system transitions between different response amplitude branches.

A subject of recent debate has been the relevance of controlled vibration to accurately predicting free vibration. The present results, with precise matching of experimental conditions, suggest that controlled vibration, even with strictly sinusoidal motion, can indeed reasonably predict free vibration responses, at least over the parameter space so far investigated.

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